Closing Fri: 3.4(part 1),
Closing Mon: 3.4(part 2)
Closing Wed: 10.2
Exam 1 will be returned Tuesday.
Grades will be posted by the end of next week.

3.4 Chain Rule

The **composition** of two function is defined by

$$(f \circ g)(x) = f(g(x))$$

Example:

If f(x) = sin(x), g(x) = x^3, then

$$(f \circ g)(x) = f(g(x)) = \sin(x^3).$$

Chain Rule:

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$$

Also written as: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Example:

$$\frac{d}{dx}\sin(x^3) = \cos(x^3)\, 3x^2$$

Here is a brief “proof sketch” for the chain rule:

From the definition of derivative

$$\begin{aligned}
\frac{d}{dx} f\big(g(x)\big) &= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h\to 0} \left(\frac{f(g(x+h)) - f(g(x))}{h} \, \frac{g(x+h) - g(x)}{g(x+h) - g(x)} \right) \\
&= \lim_{h\to 0} \left(\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \right) \left(\frac{g(x+h) - g(x)}{h} \right) \\
&= f'\big(g(x)\big) g'(x)
\end{aligned}$$

Examples: Find the derivative

1. $y = (2x^2 + 1)^2$

2. $y = e^{\sin((2x+1)^3)}$

3. $y = \tan(3x + \cos(4x))$

4. $y = \sin^4(x)$

5. $y = \sin(x^4)$

Identify the “first” rule you would use to differentiate these functions:
(sum, product, quotient or chain?)

1. $y = \sqrt{\sin(x) + x^2 + 1}$

2. $y = \dfrac{x^4}{\sin(5x+1)}$

3. $y = \sqrt[3]{4x+1}\cos(\sin(2x))$

4. $y = e^{\tan(x)} - 5(x^8+1)^{50}$

5. $y = \left(\dfrac{x^2-1}{x^4+1}\right)^{10}$